Q's 11b, 16b, 19 New additions.

Trinity College

Semester One Examination, 2016

Question/Answer Booklet

MATHEMATICS SPECIALIST UNIT 1

Section Two: Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

Student Number: In figures

In words SOLNS

Your name

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for section: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	~~49~~ 52	35
Section Two: Calculator-assumed	13	13	100	102	65
			Total	~~151~~ 154	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

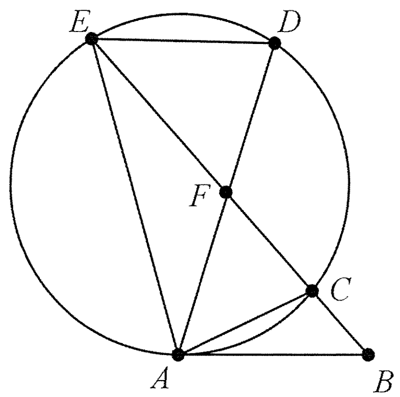
Section Two: Calculator-assumed **65% (102 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 8 **(5 marks)**

In the diagram below, AB is a tangent to the circle and $AB = BF = FD$.
Triangle AEB is similar to triangle CAB
Triangle FDE is similar to triangle FCA



If $BC = 4$ and $CE = 32$, determine the lengths of

(a) AB. (2 marks)

$AB^2 = BC \times BE$
$= 4 \times 36$ $\therefore AB = 12$

(b) CF. (1 mark)

$= 12 - 4 = 8$

(c) AF. (2 marks)

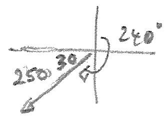
$AF \times FD = CF \times EF$
$12AF = 8 \times 24$
$AF = 16$

Question 9 **(7 marks)**

Two forces act on body. The first has magnitude 250 N and acts in direction 240° and the second has magnitude 410 N and acts in direction 170°.

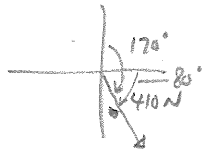
(a) Determine the resultant of the two forces. (4 marks)

F_1 :



$\begin{pmatrix} -250\cos 30° \\ -250\sin 30° \end{pmatrix} = \begin{pmatrix} -216.51 \\ -125 \end{pmatrix}$

F_2 :

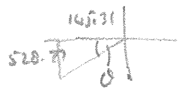


$\begin{pmatrix} 410\cos 80° \\ -410\sin 80° \end{pmatrix} = \begin{pmatrix} 71.20 \\ -403.77 \end{pmatrix}$

$\underset{\sim}{R} = \begin{pmatrix} -250\cos 30 \\ -250\sin 30 \end{pmatrix} + \begin{pmatrix} 410\cos 80 \\ -410\sin 80 \end{pmatrix}$

$= \begin{pmatrix} -145.31 \\ -528.77 \end{pmatrix}$

accept.



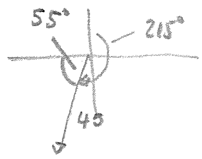
$|\underset{\sim}{R}| = \underline{548.4\ N}$

$\theta = \tan^{-1}\left(\frac{528.77}{145.31}\right)$

$= 74.63$

$\therefore$ bearing $= \underline{195.4°}$

(b) The work done, in joules, by a force in moving a body is the scalar product of the force, in newtons, and the displacement, in metres. Determine the total work done by the two forces, ~~to the nearest 100 joules~~, if the body moves 45 metres in direction 215°. (3 marks)



disp vector : $\begin{pmatrix} -45\cos 55° \\ -45\sin 55° \end{pmatrix} = \begin{pmatrix} -25.81 \\ -36.86 \end{pmatrix}$

work done $= \begin{pmatrix} -45\cos 55° \\ -45\sin 55 \end{pmatrix} \cdot \begin{pmatrix} -145.31 \\ -528.77 \end{pmatrix}$

$= 23242\ J$

or

$W = \underset{\sim}{F} \cdot \underset{\sim}{d}$

$= |\underset{\sim}{F}||\underset{\sim}{d}|\cos\theta$

$= 548.4 \times 45 \cos(215 - 195.4)$

$= 548.4 \times 45 \cos 19.6°$

$\approx \underline{23248\ J}$

Question 10 **(8 marks)**

(a) Use a counterexample to demonstrate that each of following statements are false.

(i) $ab = ac, \; \{a, b, c \in \mathbb{R}\} \; \Rightarrow \; b = c$. (2 marks)

$a = 0$, $b = 1$, $c = 2$

$0 = ab = ac$ but $b \neq c$

(ii) If $f(n) = n^2 + n + 17, \; n \in \mathbb{N}$, then $f(n)$ is always prime. (2 marks)

on CAS $f(17) = 323 = 17(19)$ $\therefore$ not prime

[isPrime]

(b) The statement 'if a natural number is a multiple of 4 and 5 then the natural number is a multiple of 20' is true.

(i) Write the contrapositive of the statement and explain whether or not the contrapositive is also true. (2 marks)

If a natural # is not a multiple of 20 then it is not a multiple of 4 and 5.

True

(a contrapositive of a true statement is always true)

(ii) Write the converse of the statement and explain whether or not the converse is also true. (2 marks)

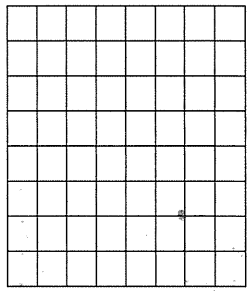
If a natural # is a multiple of 20 then it is a multiple of 4 and 5

True : as all multiples of 20 have 4 + 5 as factors.

Question 11 **(9 marks)**

(a) (i) How many rectangles of all sizes are there on an 8×8 chess board? (2 marks)

$\binom{9}{2} \times \binom{9}{2} = \underline{1296}$



9

(ii) How many of these rectangles are squares? (3 marks)

8×8 — 1
7×7 — 4
6×6 — 9
⋮
1×1 — 64

total $= 1 + 4 + 9 + \cdots + 64$
$= \underline{204}$

(b) (i) How many 4-digit numbers can be formed from the digits 1, 2, 3, 4, 5, 6, 7 and 9 if no digit may be repeated and the numbers include 2 odd digits and 2 even digits? (2 marks)

$\underbrace{\binom{5}{2}}_{\text{odd}} \times \underbrace{\binom{3}{2}}_{\text{even}} \times 4! = 720.$

(ii) How many such numbers are even? (2 marks)

$\underbrace{\binom{3}{2}}_{\text{even}}\underbrace{\binom{5}{2}}_{\text{odd}} \times 2 \times 3! = 360$

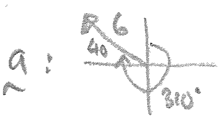
even
_ _ _ 2.

Question 12 **(7 marks)**

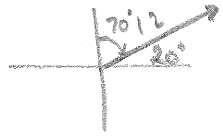
Vector $\mathbf{a}$ has magnitude 6 units and acts on a bearing of 310°. Vector $\mathbf{b}$ has magnitude 12 units and acts on a bearing of 070°.

(a) Determine the magnitude and direction of $3\mathbf{a}+2\mathbf{b}$. (4 marks)

$\underset{\sim}{a}$:



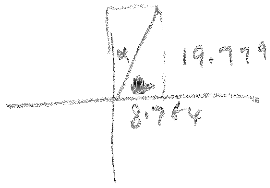
$\underset{\sim}{b}$:



$$\underset{\sim}{a} = \begin{pmatrix} -6\cos 40^\circ \\ 6\sin 40^\circ \end{pmatrix} = \begin{pmatrix} -4.6 \\ 3.9 \end{pmatrix}$$

$$\underset{\sim}{b} = \begin{pmatrix} 12\cos 20^\circ \\ 12\sin 20^\circ \end{pmatrix} = \begin{pmatrix} 11.3 \\ 4.10 \end{pmatrix}$$

$$3\underset{\sim}{a} + 2\underset{\sim}{b} = \begin{pmatrix} 8.764 \\ 19.779 \end{pmatrix}$$



$$|3\underset{\sim}{a} + 2\underset{\sim}{b}| = 21.6$$

$$\text{bearing} = \tan^{-1}\left(\frac{8.764}{19.779}\right)$$

$$= \underline{023.9^\circ}$$

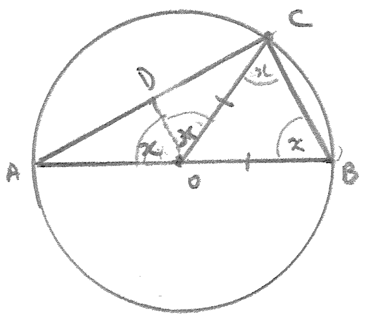
(b) Determine the value of the constant k if the direction of $5\mathbf{a}+k\mathbf{b}$ is due north. (3 marks)

$\underset{\sim}{i}$ component $= 0$ ie $-30\cos 40^\circ + 12k\cos 20^\circ = 0$

$$k = \underline{2.038}$$

Question 13 **(9 marks)**

(a) AB is a diameter of a circle centre O. C is a point on the circumference. D is a point on AC such that OD bisects $\angle AOC$. Prove that OD is parallel to BC. (4 marks)



let $\angle ABC = x^\circ$ (angle at circumference $= \frac{1}{2}$ angle at centre on minor arc AC)

Δ OBC is isosceles (OC and OB are radii)

$\therefore \angle OCB = x^\circ$

$\angle DOC = \angle OCB$ (alternate $\angle$'s)

$\therefore OD \parallel BC$

OR let $\vec{OC} = \underset{\sim}{c}$, $\vec{OA} = \underset{\sim}{a}$

since Δ OAC is isoseles (AO, CO radii) then $AD = CD$ since $\angle AOD = COD$ (given). Δ's ADO + CDO are $\cong$ SAS.

Now $\vec{BC} = \underset{\sim}{c} - (-\underset{\sim}{a}) = \underset{\sim}{a} + \underset{\sim}{c}$

$\vec{OD} = \underset{\sim}{a} + \frac{1}{2}(\underset{\sim}{c} - \underset{\sim}{a})$

$= \frac{1}{2}(\underset{\sim}{a} + \underset{\sim}{c})$

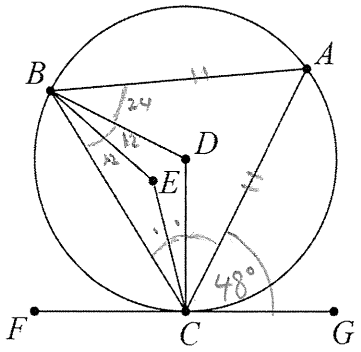
$\therefore BC \parallel OD$ since $\vec{OD}$ is a scalar multiple of $\vec{BC}$

OR

$\Rightarrow \angle ADO = 90^\circ$

$\therefore \angle$'s ODC + BCD - interior angles $\therefore BC \parallel OD$

(b) In the diagram below, FCG is a tangent to the circle ABC. BD bisects $\angle ABC$, CD bisects $\angle ACB$, BE bisects $\angle DBC$ and CE bisects $\angle DCB$. If $AB = AC$ and $\angle ACG = 48°$, determine the ratio, in simplest form, of $\angle BAC : \angle BDC : \angle BEC$. (5 marks)



$\angle ABC = 48°$ (alternate seg thm)

$\angle ACB = 48°$ (ΔABC isosceles)

$\therefore \angle BAC = 180 - 2(48)$
$= 84°$

$\angle BDC = 180 - 2(24)$
$= 132°$

$\angle BEC = 180 - 2(12)$
$= 156$

$\therefore \angle BAC : \angle BDC : \angle BEC$

$= 84 : 132 : 156$

$= 7 : 11 : 13$

Question 14 **(9 marks)**

O $\begin{pmatrix}0\\0\end{pmatrix}$ P $\begin{pmatrix}15\\y\end{pmatrix}$ Q $\begin{pmatrix}x\\-1\end{pmatrix}$ R $\begin{pmatrix}3\\-5\end{pmatrix}$

Points O, P, Q and R have position vectors $(0,0)$, $(15, y)$, $(x,-1)$ and $(2,-5)$.

3

(a) Determine the value of y if $|\overrightarrow{OP}| = 17$. (2 marks)

$15^2 + y^2 = 17^2$

$\underline{y = \pm 8}$

(b) Determine the value of x if $\overrightarrow{OQ} \perp \overrightarrow{QR}$. (3 marks)

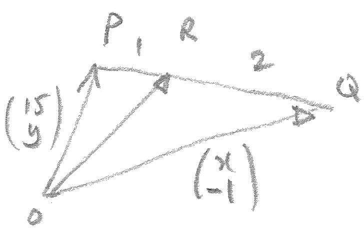
$\overrightarrow{OQ} = \begin{pmatrix}x\\-1\end{pmatrix}$ $\quad \overrightarrow{QR} = \begin{pmatrix}3-x\\-4\end{pmatrix}$

$\therefore \begin{pmatrix}x\\-1\end{pmatrix} \cdot \begin{pmatrix}3-x\\-4\end{pmatrix} = 0$

ie $3x - x^2 + 4 = 0$

$x = 4$ or -1

(c) Determine the values of x and y if R lies on the line between P and Q such that $PR : RQ = 1:2$. (4 marks)



$\overrightarrow{OR} = \begin{pmatrix}15\\y\end{pmatrix} + \frac{1}{3}\begin{pmatrix}x-15\\-1-y\end{pmatrix}$

ie $\begin{pmatrix}3\\-5\end{pmatrix} = \begin{pmatrix}15 + \frac{1}{3}x - 5\\ y - \frac{1}{3} - \frac{1}{3}y\end{pmatrix}$

$\therefore 3 = 10 + \frac{1}{3}x \quad \therefore \underline{x = -21}$

$\& \ -5 = \frac{2}{3}y - \frac{1}{3} \quad \therefore \underline{y = -7}$

$\overrightarrow{PQ} = \begin{pmatrix}x-15\\-1-y\end{pmatrix}$

Question 15 **(9 marks)**

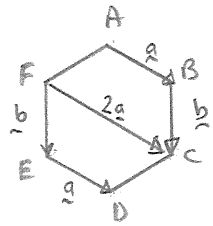
(a) $ABCDEF$ is a regular hexagon in which $\overrightarrow{BC}$ represents $\mathbf{b}$ and $\overrightarrow{FC}$ represents $2\mathbf{a}$.

Express the vectors $\overrightarrow{CD}$, $\overrightarrow{EA}$ and $\overrightarrow{BE}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. (3 marks)

$\overrightarrow{CD} = -2\underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{a} = \underline{\underset{\sim}{b} - \underset{\sim}{a}}$

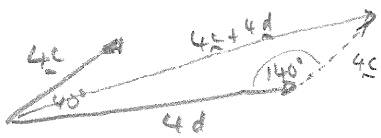
$\overrightarrow{EA} = -\underset{\sim}{b} + \underset{\sim}{a} - b = \underline{a - 2\underset{\sim}{b}}$

$\overrightarrow{BE} = \underset{\sim}{b} + \underset{\sim}{b} - \underset{\sim}{a} - \underset{\sim}{a} = \underline{2\underset{\sim}{b} - 2\underset{\sim}{a}}$



(b) Given $\mathbf{c}$ and $\mathbf{d}$ are vectors such that $|\mathbf{c}| = 4$, $|\mathbf{d}| = 6$ and the angle between their directions is 40°, determine

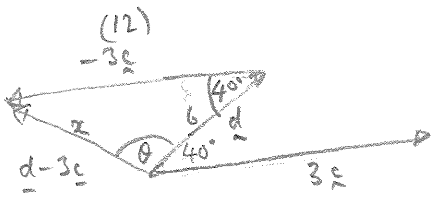
(i) $|4\mathbf{c} + 3\mathbf{d}|$. (3 marks)



cos rule: $|4\underset{\sim}{c} + 4\underset{\sim}{d}| = \sqrt{16^2 + 18^2 - 2.16.18 \cos 140°}$

$= \underline{31.96}$

(ii) the angle between $\mathbf{d}$ and $\mathbf{d} - 3\mathbf{c}$. (3 marks)



$x = \sqrt{6^2 + 12^2 - 2.6.12 \cos 40°}$

$= \underline{8.348}$

$\dfrac{12}{\sin\theta} = \dfrac{8.348}{\sin 40°}$ $\therefore \theta = 67.5$ or $\underline{112.5°}$

$\therefore \theta = 112.5°$ (as θ cannot be acute)

OR use cos rule to find θ.

$\cos\theta = \dfrac{6^2 + 8.348^2 - 12^2}{2.6.8.348}$

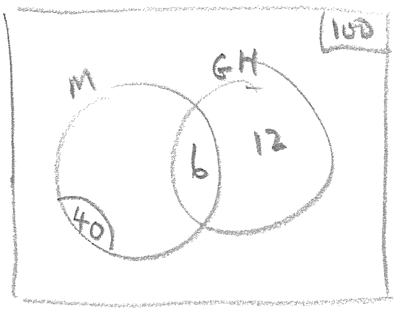
$\theta = \underline{112.5°}$

Question 16 **(10 marks)**

(a) If a room contains 75 adults, use the pigeonhole principle to explain why a group containing at least 11 of these people could be chosen so that all were born on the same day of the week. (3 marks)

$75 \div 7 = 10.71$ $\therefore$ at least 11 were born on same day by PHP.

(b) In a certain community 40% of the people are males. If 15% of the males are grey-haired and 20% of the females are grey-haired, what percentage of the grey-haired people are males? (3 marks)



$\frac{6}{18} = \frac{1}{3} = 33\frac{1}{3}\%$

	G	$\bar{G}$	
M	6	34	40
F	12	48	60
	18	82	100

$\frac{6}{18}$

(c) Determine how many numbers between 1 and 150 inclusive are multiples of 3, 4 or 5. (4 marks)

Multiples of 3 : 50
" " 4 : 37 ($150 \div 4 = 37.5$)
" " 5 : 30

" 3 & 4 : 12 ($150 \div 12 = 12.5$)
" 3 & 5 : 10
" 4 & 5 : 7 ($150 \div 20 = 7.5$)
" 3, 4 & 5 : 2 ($150 \div 60 = 2.5$)

Using pr of inclusion - exclusion

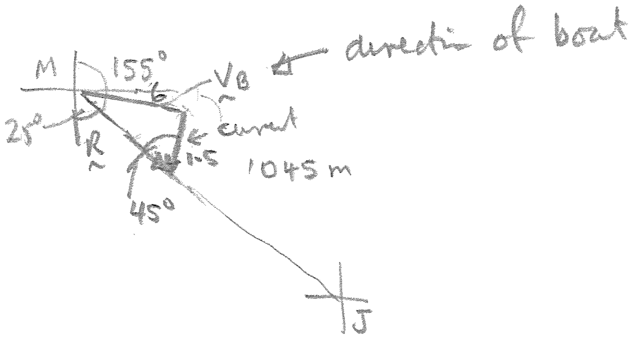
number $= 50 + 37 + 30 - 12 - 10 - 7 + 2$

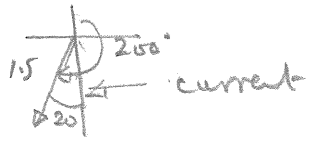
$= \underline{90}$

Question 17 **(8 marks)**

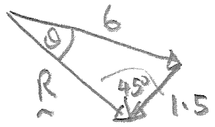
A boat with a constant speed of 6 ms^{-1} is required to leave its mooring and motor directly to a jetty located 1 045 metres away on a bearing of 155°. A current of 1.5 ms^{-1} is running on a bearing of 200°.

(a) Sketch a diagram to that can be used to determine the direction the boat should steer. (2 marks)





(b) Determine the bearing that the boat should steer. (3 marks)



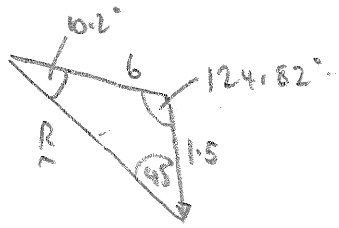
$$\frac{6}{\sin 45^\circ} = \frac{1.5}{\sin\theta}$$

$\theta = 10.2^\circ$ (10.182)

$\therefore$ bearing $= 155 - 10.2$

$= \underline{144.8^\circ}$

(c) Determine the time taken for the boat to reach the jetty. (3 marks)



cos rule gives $|R| = 6.966 \ ms^{-1}$

$\therefore$ time $= \dfrac{1045}{6.966} \approx \underline{\underline{150 \ s}}$

Question 18 **(7 marks)**

(a) Let $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j}$, $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j}$ and $\mathbf{c} = c_1\mathbf{i} + c_2\mathbf{j}$.

Prove that the scalar product is distributive over vector addition: $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$.

(4 marks)

$LHS = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \cdot \begin{pmatrix} b_1 + c_1 \\ b_2 + c_2 \end{pmatrix} = a_1(b_1 + c_1) + a_2(b_2 + c_2)$

$RHS = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} + \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \cdot \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$

$= a_1 b_1 + a_2 b_2 + a_1 c_1 + a_2 c_2$

$= a_1(b_1 + c_1) + a_2(b_2 + c_2) = \underline{LHS}$

(b) Given $\mathbf{p} = 1.5\mathbf{i} + 4.5\mathbf{j}$, $\mathbf{q} = 3.5\mathbf{i} - 1.5\mathbf{j}$, $\mathbf{r} = -3.5\mathbf{i} + 5.5\mathbf{j}$ and $\mathbf{s} = -1.5\mathbf{i} + 1.5\mathbf{j}$, determine $\mathbf{p} \cdot \mathbf{r} - \mathbf{p} \cdot \mathbf{s} + \mathbf{q} \cdot \mathbf{r} - \mathbf{q} \cdot \mathbf{s}$.

(3 marks)

$= \underset{\sim}{p} \cdot (\underset{\sim}{r} - \underset{\sim}{s}) + \underset{\sim}{q} \cdot (\underset{\sim}{r} - \underset{\sim}{s})$

$= (\underset{\sim}{p} + \underset{\sim}{q}) \cdot (\underset{\sim}{r} - \underset{\sim}{s})$

$= \begin{pmatrix} 5 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 4 \end{pmatrix} = -10 + 12$

$= \underline{2}$

Question 19 **(8 marks)**

A bag contains ten discs, three red, two green and one each of blue, yellow, black, white and brown.

(a) Determine the total number of selections there are of four of these discs. (Note: there are 5 different ways of selecting 4 discs from the bag) (4 marks)

① 4 different : $\binom{7}{4} = 35$

② 2R + 2 : $1 \times \binom{6}{2} = 15$

③ 2G + 2 : $1 \times \binom{6}{2} = 15$

④ 2R + 2G : $1 = 1$

⑤ 3R + 1 : $1 \times \binom{6}{1} = 6$

sum = 72

(c) Determine the total number of arrangements there are of four of these discs. (4 marks)

① $35 \times 4!$

② $\frac{15 \times 4!}{2!}$

③ $\frac{15 \times 4!}{2!}$

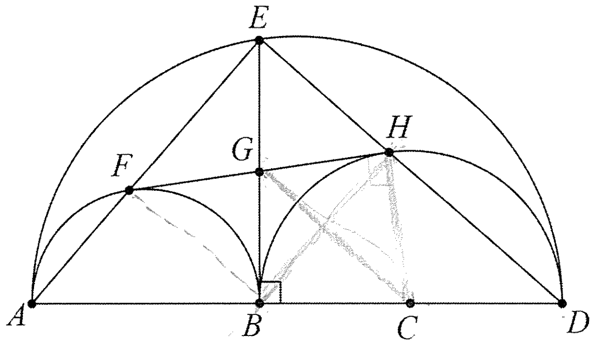
④ $\frac{4!}{2!\,2!}$

⑤ $6 \times \frac{4!}{3!}$

Sum = 1230

Question 20 **(7 marks)**

The diagram shows three semicircles with diameters AD, AB and BD, where B is a point on the diameter AD. Point C is the centre of the semicircle with diameter BD. Line BE is perpendicular to diameter AD and meets the largest semicircle at E. Points F and H are the intersections of lines AE and DE with the smaller semicircles. Point G is the intersection of lines FH and BE.



(a) Explain why $BFEH$ is a rectangle. (2 marks)

$\angle AED = 90°$ (angle in semicircle AED)
$\angle$'s AFB = $\angle$DHB = 90° (angles in semicircle AFB & DHB)
$\therefore \angle$'s BFE & BHE = 90° (supplementary angles).
$\therefore$ rectangle

(b) Prove that $\triangle CBG$ and $\triangle CHG$ are congruent. (3 marks)

CH = BC (radii of same circle)
GC (common)
BG = HG (each ½ diagonal of rectangle BFEH)

$\therefore \triangle CBG \cong \triangle CHG$ (SSS)

(c) Deduce that line FH is a tangent to the semicircle with diameter BD. (2 marks)

$\angle CBG = \angle CHG$ (corresponding angles of congruent $\triangle$s)
= 90° (given)

$\therefore FH \perp CH$ $\therefore$ FH is a tangent (CH radius).

End of questions